



# Improving an efficient simulation of multidimensional Gaussian random fields with GPU

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# Context

#### **Resource Assesment**

- Estimation Kriging
- Simulation
  *Turning Bands Method*





#### **Turning Bands Method**

Turning bands simplifies the complexity of the simulation with a dimensional reduction, by computing simulated values over lines, that is reducing the problem to one-dimensional simulations, and projecting them to the three-dimensional space, thereby allowing the use of very efficient simulation methods to obtain the one-dimensional simulations.

$$C_1(r) = \frac{d}{dr} [r C_3(r)]$$









### **Turning Bands Method**

The method Proceeds in the following fashion:

- 1. Simulated values  $X_i$  are generated over lines randomly oriented over the unit sphere, with covariance function  $C_x$
- 2. For every location on the three dimensional simulated grid, the simulated values generated over the line are projected orthogonally into the simulated location.
- 3. The final simulated value is obtained as the projection of values frommany lines:  $1 \quad \sum_{n=1}^{N}$

$$Y(x) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} X_i(\langle x, \mathbf{U}_i \rangle),$$

where  $U_i$  are lines randomly distributed over the sphere  $S_3$ ,  $X_i$  is a random value with covariance function  $C_x$  and x is the location where we want to simulate.

4. Conditioning to the actual samples is done as a postprocess through the method of kriging residuals. Let  $\{Y_s(x), x \in \mathbb{R}^3\}$  be a non-conditional simulation. The random defined by

 $\forall x \in \mathbf{R}^3, \ Y_{CS}(x) = Y_S(x) + [Y(x) - Y_S(x)]^{SK}$ 

represent the conditional simulation that reproduce the distribution of  $\{Y(x), x \in \mathbb{R}^3\}$  conditional to the Y-data.

#### The timeout problem and our proposal

- The simulation size is too large --> this method has a low performance within CPU
- Low performance implies very long processing times.
- For large resources models, computation times may be impractical.
- We use the GPU to parallelize the simulation process in the Turning Bands Method and thereby reduce the timeout.



#### Implementation details

- 90% of processing time in GPU is devoted to simulation computations and the other 10% is related to data transfer between GPU and CPU.
- Processing Power over 400.000 Grid Points per Second
- Unlimited grid size simulation
- The parallel algorithm involves over 98% of the computational cost involved in the serial implementation.
- There is only two data transfer between CPU and GPU memory.





### Performance

	CPU	GPU
Model Name	Intel Xeon E5405	Tesla C2050
$Number\ of\ Cores$	4	448
Frequency of Cores	$2.67 \mathrm{GHz}$	$1.15 \mathrm{GHz}$
$RAM\ memory$	$4\mathrm{GB}$	3GB

Serial and Parallel execution times are measured for runs of the implementations under different settings:

- Varying the simulated grid size from 1000 to 20 000 000 nodes.
- Running 10 to 100 realizations





#### Performance CPU vs Realizations (Serial Implementation)





#### Performance GPU vs Grid Points (Parallel implementation)





#### Performance CPU vs Realizations (Serial Implementation)





#### Performance GPU vs Realizations (Parallel implementation)





# Summary of Performance

SpeedUp vs	Grid Points	Realizations	Processing Power	CPU	GPU
Average	36,4x	43,2x	Average	11.560	400.061
Standard Deviation	2,1x	1,3x	Standard Deviation	518	17.171



### Example

- Three different grid sizes were simulated and conditionated with a real data base.
- The data base contains 2375 samples in a copper deposit to condition the simulation process.





	CPU Time	GPU Time	$\mathbf{SpeedUp}$
realizations = 100	[hour]	[minute]	
N = 1000000	$2,\!5$	$6,\!8$	21,5x
N = 4000000	$9,\!8$	$20,\!8$	28,2x
N = 8000000	19,5	$34,\!8$	$33,\!6x$

## Conclusions

- We have shown the application of a parallelized implementation of the turning bands simulation method using GPU.
- The optimum speedup is achieved when a large number of realizations is required.
- In a practical application, speedup reached **33x.**



 Comparisons were also made against public domain code (sgsim from GSLIB); speedup in this case reached *100x* (not shown here).

